[Section - A]

1. (A) No Solution

2. (D) $\frac{4}{\sqrt{15}}$

3. (D) Irrational Number

4. (C) 0

5. (A) $x^2 + 2x = 0$

6. (A) 2

7. (C) b

8. (C) 3 cm.

9. (C) 150°

10. (C) $\frac{3}{4}$

11. (D) 6 cm

12. (C) $x < y$

13. (B) Q

14. (B) 45°

15. (A) 30°

16 (C) 2 : 1

17. (B) 13 and 12

18. (C) 52 is the mode of the data.

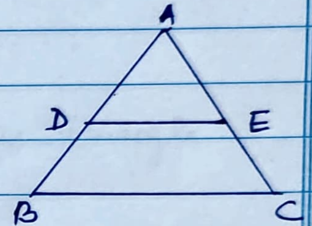
19. (D) Assertion (A) is false, but reason (R) is true.

20. (D) Assertion (A) is false, but reason (R) is true.

(Section – B)

Solu:-

(21) <u>Given</u> :- $\frac{AD}{BD} = \frac{AE}{EC}$ & $\angle BDE = \angle CED$



<u>To prove</u>:- ΔABC is an isosceles triangle.

<u>Proof</u> :- In ΔADE & ~~ΔABC~~

$$\frac{AD}{BD} = \frac{AE}{EC}$$

by Converse of B.P.T :-

DE ∥ BC

$\therefore$ $\angle ADE = \angle ABC$ —① & $\angle AED = \angle ACB$ (Corresponding ∠s) —②

<u>Now</u>,

$\angle ADE + \angle BDE = 180$ (linear pair)

$\angle BDE = 180 - \angle ADE$

& $\angle AED + \cancel{\angle ACB}\ \angle CED = 180$

$\angle CED = 180 - \angle AED$

$\therefore$ $180 - \angle ADE = 180 - \angle AED$ ($\angle BDE = \angle CED$)

$\angle ADE = \angle AED$

$\therefore$ from eq ① & ②

$\angle ABC = \angle ACB$

Hence it is proved that ΔABC is an isosceles triangle.

(22) i) $P(\text{Perfect square}) = \frac{8}{96} = \frac{1}{12}$

ii) $P(\text{2 digit number}) = \frac{91}{96}$ ans.

(23) a) $101x + 102y = 304$ —— ①

$102x + 101y = 305$ —— ②

Adding eq. ① + ② :-

$203x + 203y = 609$

$x + y = 3$ —— ③

Subtracting eq. ① from ② :-

$x - y = 1$ —— ④

Add eq. 3 & 4

$$2x = 4$$

$$\boxed{x = 2}$$

Subtitute, $x = 2$ in eq. 3

$$\boxed{y = 1}$$

23 b) Let two supplementary angles be x & y

A.t.q.

$$x + y = 180^\circ \quad \text{—— (1)}$$

& $$x = y + 50 \quad \text{—— (2)}$$

From eq. 2 & 1 :-

$$y + 50 + y = 180$$

$$2y = 130$$

$$\boxed{y = 65^\circ}$$

Substitute $y = 65$ in eq. 2

$$\boxed{x = 115^\circ}$$

(24) (a) Given :- $a \sec\theta + b\tan\theta = m$ — (1)

$b\sec\theta + a\tan\theta = n$ — (2)

To prove :-

$$a^2 + n^2 = b^2 + m^2$$

Proof :-

Subtracting square of eq. (2) from square of eq. (1)

$$a^2\sec^2\theta + b^2\tan^2\theta = m^2$$

$$\underline{-b^2\sec^2\theta \pm a^2\tan^2\theta = -n^2}$$

$$\sec^2\theta\,(a^2 - b^2) + \tan^2\theta\,(b^2 - a^2) = m^2 - n^2$$

$$(a^2 - b^2)(\sec^2\theta - \tan^2\theta) = m^2 - n^2$$

$$a^2 - b^2 \times 1 = m^2 - n^2$$

$$\boxed{a^2 + n^2 = b^2 + m^2}$$ proved

(24.) b) L.H.S

$= \tan^2 A + 1$

$= \dfrac{\sin^2 A}{\cos^2 A} + 1 = \dfrac{\sin^2 A + \cos^2 A}{\cos^2 A} = \dfrac{1}{\cos^2 A} = \sec^2 A$ = R.H.S

Now,

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A + 1 = \left(\frac{5}{3}\right)^2$

$\tan^2 A = \frac{25}{9} - 1$

$\tan^2 A = \frac{16}{9}$

$\tan A = \sqrt{\frac{16}{9}}$

$\tan A = \pm \frac{4}{3}$

A is an acute angle

$\boxed{\therefore \tan A = \frac{4}{3}}$

(25) To prove:- $x = y + 2$

Proof:-

$$PA = PB$$

$$\sqrt{(x-7)^2 + (y-1)^2} = \sqrt{(x-3)^2 + (y-5)^2}$$

$$x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$$

$$50 - 14x - 2y = 34 - 6x - 10y$$

$$16 + 8y = 8x$$

$$8(2+y) = 8x$$

$$2 + y = x$$

$$\boxed{x = y + 2}$$ proved

(26) a) L.H.S.

$$= \frac{\cos\theta - 2\cos^3\theta}{\sin\theta - 2\sin^3\theta} + \cot\theta \Rightarrow \frac{\cos\theta(1-2\cos^2\theta)}{\sin\theta(1-\sin^2\theta)} + \cot\theta$$

$$= \cot\theta \frac{(1-2\cos^2\theta)}{(1-2\sin^2\theta)} + \cot\theta$$

$$= \cot\theta \left[\frac{1-2(1-\sin^2\theta)}{1-2\sin^2\theta}\right] + \cot\theta$$

$$= \cot\theta \left(\frac{-1+2\sin^2\theta}{1-2\sin^2\theta}\right) + \cot\theta$$

$$= -\cot\theta \left(\frac{1-2\sin^2\theta}{1-2\sin^2\theta}\right) + \cot\theta$$

$$= -\cot\theta + \cot\theta = 0 = R.H.S$$

proved

(26) b) Given :- $\sin\theta + \cos\theta = x$

To Prove :- $\sin^4\theta + \cos^4\theta = \dfrac{2-(x^2-1)^2}{2}$

Proof :-

$$\sin\theta + \cos\theta = x$$

Squaring both sides -

$$(\sin\theta + \cos\theta)^2 = x^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = x^2$$

$$1 + 2\sin\theta\cos\theta = x^2$$

$$2\sin\theta\cos\theta = x^2 - 1$$

Again, squaring both sides -

$$4\sin^2\theta\cos^2\theta = (x^2-1)^2$$

diving both sides by 2 :-

$$2\sin^2\theta\cos^2\theta = \frac{(x^2-1)^2}{2} \quad \text{--- } (1)$$

Now,

We know that,

$$\sin^2\theta + \cos^2\theta = 1$$

Squaring both sides :-

$$(\sin^2\theta + \cos^2\theta)^2 = 1^2$$

$$\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$$

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$$

$$\sin^4\theta + \cos^4\theta = 1 - \frac{(x^2-1)^2}{2} \quad (\text{From eq. } 1)$$

$$\sin^4\theta + \cos^4\theta = \frac{2-(x^2-1)^2}{2}$$

Proved

(27) We have,

$\angle OPQ + \angle TPQ = 90^\circ$ [tangents are perpendicular to radius at point of contact]

$\angle TPQ = 90 - 15$

$\angle TPQ = 75^\circ$

Also, $\angle TPQ = \angle TQP = 75^\circ$ [TP = TQ, length of tangents]

Now,

In ΔTPQ,

$75^\circ + 75^\circ + \theta = 180^\circ$ (Angle sum property)

$\boxed{\theta = 30^\circ}$

So, $\sin 2\theta = \sin 30 \times 2 = \sin 60^\circ = \boxed{\frac{\sqrt{3}}{2}}$ ans.

(28) (a) let $\sqrt{5}$ is a rational number

$\therefore \sqrt{5} = \frac{a}{b}$ [a & b are Co-prime & $b \neq 0$]

$\sqrt{5}b = a$

Squaring both sides,

$(\sqrt{5}b)^2 = a^2$

$5b^2 = a^2$ — (1)

Hence 5 is a factor of a^2

So 5 is also a factor of a. — (2)

Now,

let, $5c = a$

Subtitute the value of a in eq.(1)

$5b^2 = (5c)^2$

$5b^2 = 25c^2$

$b^2 = 5c^2$

Hence 5 is a factor of b^2

So 5 is also a factor of b. — (3)

from (2) & (3)

5 is common factor of a & b

So a & 5 are not co-prime number

Hence, Our assumption is wrong

$\therefore$ by contradiction, $\sqrt{5}$ is irrational.

Proved

(28) b) We have,

$$p.q.r + q = q(p.r + 1)$$

So, $p.q.r + q$ divided by q

Hence it is a composite number.

i) let, $p = 3$, $q = 5$, $r = 7$

$pqr + 1 = 3 \times 5 \times 7 + 1 = 106$

it is divisible by 2 so it is a composite number.

ii) Let, $p = 2,\ q = 3,\ r = 5$

$p.q.r + 1 = 2 \times 3 \times 5 + 1 = 31$

So it is a prime number.

(29) We have,

$$8x^2 - 2x - 3 = 0$$
$$8x^2 - 6x + 4x - 3 = 0$$
$$2x(4x-3) + 1(4x-3) = 0$$
$$(4x-3)(2x+1) = 0$$

So, $\boxed{x = \frac{3}{4} \ \& \ x = -\frac{1}{2}}$

Now, New zeroes,

$\alpha = \frac{3}{4} - 2 = \frac{-5}{4}$ & $\beta = -\frac{1}{2} - 2 = \frac{-5}{2}$

$\alpha + \beta = -\frac{5}{4} - \frac{5}{2} = \frac{-15}{4}$

$\alpha.\beta = -5/4 \times -5/2 = 25/8$

So Polynomial $= x^2 - (\alpha+\beta)x + \alpha\beta = x^2 + \frac{15}{4}x + \frac{25}{8} = \boxed{\cancel{4x^2 + 15x + 14}}$

$\boxed{8x^2 + 30x + 25}$

af.

(80) $x - 2y + 4 = 0$ —— (1)

$2x - y - 4 = 0$ —— (2)

$\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = -\frac{2}{1}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Unique solution)

Hence it Consistent.

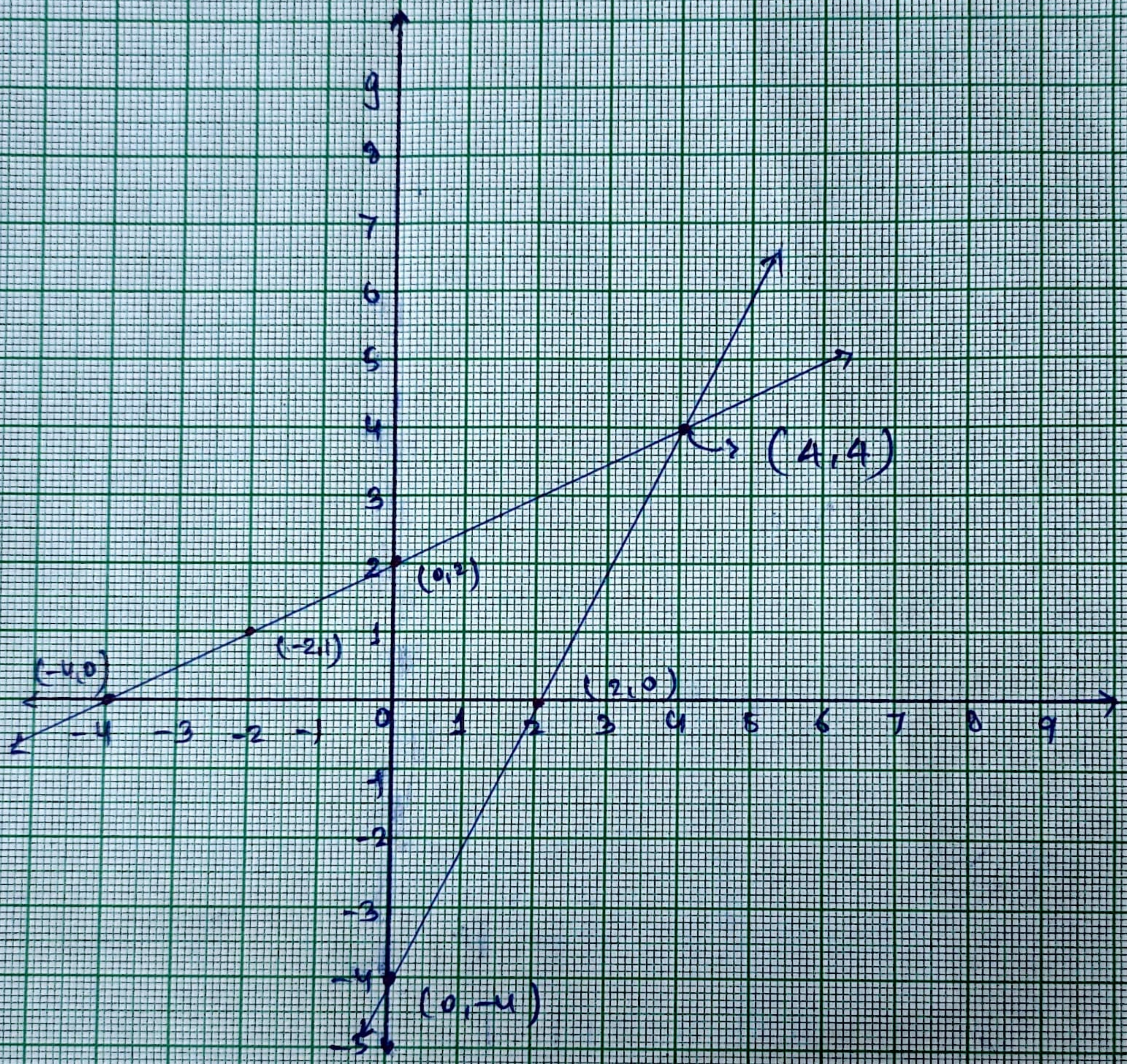
Now, From eq (1)

x	0	-4	-2
y	2	0	1

From eq. (2)

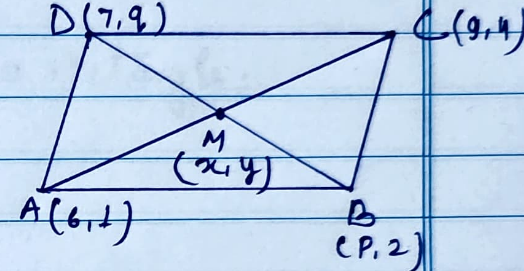
x	0	2
y	-4	0

So $x = 4$ & $y = 4$ is the solution.


(4,4)
(0,2)
(-2,1)
(-4,0)
(2,0)
(0,-4)

(31) We have, A(6,1) B(P,2) C(9,4) D(7,9) are vertice of a ||gm.

We know that, diagonal of ||gm bisect each other.



So, M is Mid point of AC & BD

Now,

$$x = \frac{6+9}{2} = \frac{7+P}{2}$$

$$15 - 7 = P$$

$$\boxed{P = 8}$$

$$y = \frac{9+2}{2} = \frac{1+4}{2}$$

$$\boxed{9 = 3}$$

Now, B(8,2) & D(7,3)

Diagonal AC $= \sqrt{3^2+3^2} = 3\sqrt{2}$

Diagonal BD $= \sqrt{1^2+1^2} = \sqrt{2}$

$AC \neq BD$, So it is not a rectangle.

(32)

Number of member	N. of Bunglows	c.f.
0 - 2	10	10
2 - 4	p	10 + p
4 - 6	60	70 + p
6 - 8	q	70 + p + q
8 - 10	5	75 + p + q
	Total = 120	

Median Class = 4 - 6

$f = 60$

$70 + p + q = 120$

$p + q = 50$ — ①

Median = 5

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f} \times h\right] = 4 + \left(\frac{60 - 10 - p}{60} \times 2\right)$$

$$5 = 4 + \left(\frac{50 - p}{30}\right)$$

$$1 = \frac{50 - p}{30}$$

$$\boxed{p = 20}$$

Substitute the value in eq. ①

$$\boxed{q = 30}$$

ans.

(33) let $PB = x$

$PA = 35 + x$

In right $\triangle ABP$,

$AB^2 = x^2 + (35+x^2)$

$65^2 = x^2 + 1225 + x^2 + 35x$

$4225 = 2x^2 + 35x + 1225$

$x^2 + 35x - 1500 = 0$

Now,

$D = b^2 - 4ac$

$= 35^2 - 4 \times 1 \times (-1500)$

$= 7225$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-35 \pm \sqrt{7225}}{2 \times 1} = \frac{-35 \pm 85}{2}$

$\boxed{x = 25\,m}$

So Distance of P from B = 25 m

& P from A = 25 + 35 = 60 m

of.

b) given equation,

$$x^2 - 2(p+1)x + p^2 = 0$$

Here, $a = x^2$
$b = -2(p+1)$
$c = p^2$

For real roots,

$$D \geq 0$$

$$b^2 - 4ac \geq 0$$

$$[-2(p+1)]^2 - 4 \times 1 \times p^2 \geq 0$$

$$4(p+1)^2 - 4p^2 \geq 0$$

$$4[(p+1)^2 - p^2] \geq 0$$

$$p^2 + 1^2 + 2p - p^2 \geq 0$$

$$1 + 2p \geq 0$$

$$p \geq -\frac{1}{2}$$

So, Smallest value of $p = -\frac{1}{2}$

ans.

(34) We have,

$r = 2.1$ mm

$h = 2.8$ mm

$l = \sqrt{r^2+h^2} = \sqrt{(2.1)^2+(2.8)^2} = \sqrt{12.25} = 3.5$ mm

T.S.A of pencil = 2×C.S.A of Cone + C.S.A of Cylinder

$= 2\pi rl + 2\pi rh = 2\pi r(l+h)$

$= 2 \times \frac{22}{7} \times 2.1 \times (3.5 + 100)$

$= 13.2 \times 103.5$

$= \boxed{1366.2 \text{ mm}^2}$ ans.

(35) We have,

$h = 14$ cm

$r = 7$ cm

$l = \sqrt{r^2+h^2} = \sqrt{7^2+14^2} = \sqrt{245} = 7\sqrt{5}$

$l = 7 \times 2.2 = 15.4$ cm

C.S.A of remaing solid = T.S.A of cube $- \pi r^2 +$ C.S.A of cone

$= 6a^2 - \pi r^2 + \pi r l$

$= 6 \times 14 \times 14 - \pi r(r - l)$

$= 1176 - \frac{22}{7} \times 7 (7 - 15.4)$

$= 1176 - 22 \times (-8.4)$

$= 1176 + 184.8 = \boxed{1360.8 \text{ cm}^2}$

Hence C.S.A of remaining solid is 1360.8 cm^2

(36)

i) $a = 400\ m$

$d = 7.6\ m$

$t_6 = a + 5d = 400 + 5 \times 7.6 = \boxed{438\ m}$

ii) $t_8 - t_4$

$= a + 7d - (a + 3d)$

$= a + 7d - a - 3d$

$= 4d = 4 \times 7.6 = \boxed{30.4\ m}$

iii)

a) $S_6 = \frac{n}{2}[2a + (n-1)d] = \frac{6}{2}[2 \times 400 + 5 \times 7.6]$

$= 3(800 + 38) = 3 \times 838 = \boxed{2514\ m}$ Af.

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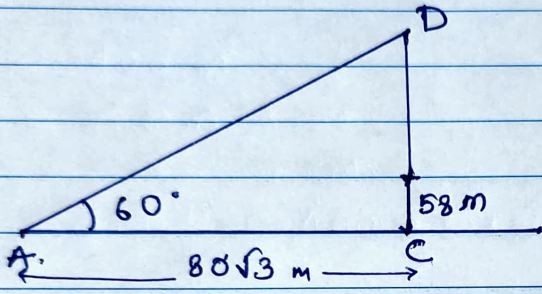
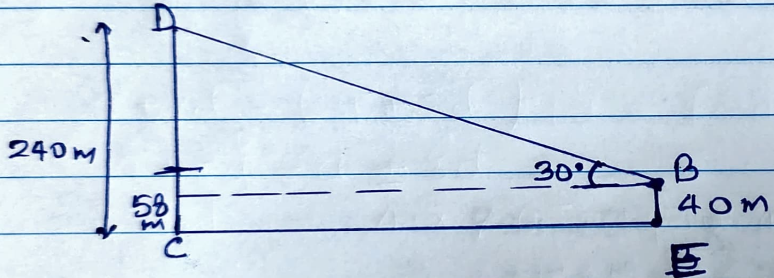
i)



ii)



iii) a) Height of statue excluding base $= 240 - 58 = 182$ m

A.t. situation I, height of statue $= 240$ m.

(38)

i) In ΔAPO,

$AO = PO$ (radius)

$\therefore \angle PAO = \angle APO = 30^\circ$

Now,

$\angle POA + \angle PAO + \angle APO = 180^\circ$

$\angle POA = 180 - 60$

$\boxed{\angle POA = 120^\circ}$

ii) length of wire = Perimeter of semicircle + diameter

$= \pi r + 2r + 70$

$= 3.14 \times 35 + 2 \times 35 + 70$

$= 109.9 + 70 = \boxed{179.9\ m}$

iii) Area $= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times 3.14 \times 35^2 = \frac{3.14 \times 1225}{6}$

$= \frac{3843.5}{6} = \boxed{640.58\ m^2}$

a.